

In a nutshell: Successive parabolic interpolation

Given a continuous and differentiable real-valued function f of a real variable with three initial approximation of an extremum x_{-2}, x_{-1}, x_0 . This algorithm uses iteration and interpolating polynomials.

Parameters:

- ϵ_{step} The maximum error in the value of the root cannot exceed this value.
- ϵ_{abs} The value of the function at the approximation of the root cannot exceed this value.
- N The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
3. Reorder the points x_{k-2}, x_{k-1} and x_k so that $f(x_{k-2}) \geq f(x_{k-1}) \geq f(x_k)$, and if two are equal, try to ensure that the one in the middle is x_k .
4. The next approximation to the extremum will be the minimum of the interpolating polynomial that passes through the points $(x_{k-2}, f(x_{k-2})), (x_{k-1}, f(x_{k-1})), (x_k, f(x_k))$.

$$\text{Let } x_{k+1} \leftarrow \frac{x_k + x_{k-1}}{2} + \frac{1}{2} \frac{(f(x_k) - f(x_{k-1}))(x_{k-1} - x_{k-2})(x_{k-2} - x_k)}{(f(x_k) - f(x_{k-1}))(x_{k-1} - x_{k-2}) + (f(x_{k-2}) - f(x_{k-1}))(x_k - x_{k-1})}.$$

- a. If $f(x_{k+1}) \geq f(x_{k-2})$, there is an issue with this method, so return signalling a failure to converge.
 - b. If x_{k+1} is not a finite floating-point number, so return signalling a failure to converge.
 - c. If $|x_{k+1} - x_k| < \epsilon_{\text{step}}$ and $|f(x_{k+1}) - f(x_k)| < \epsilon_{\text{abs}}$, return x_{k+1} .
5. Increment k and return to Step 2.

If this method converges, then if $f^{(2)}(x_{k+1}) > 0$, it is a minimum; if $f^{(2)}(x_{k+1}) < 0$, it is a maximum; but if $f^{(2)}(x_{k+1}) \approx 0$, it could be a maximum, a minimum, or a saddle point.

Convergence

If h is the error, it can be show that the error decreases according to $O(h^{1.3247})$ where the exponent is the real root of $x^3 - x - 1$. This technique is not guaranteed to converge if there is a root, for the points are close to being collinear, causing the next approximation to be arbitrarily far from the previous approximation.